Teaching and Educational Methods

Calculating and Interpreting Percentage Changes for Economic Analysis

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Abstract
A strong understanding of calculating and interpreting percentage changes and growth rates is critically important for economists. This is because many fundamental concepts such as the time value of money, and many commonly reported economic measures, such as the rate of return on assets, price inflation, and measures of economic growth, require a firm understanding of percentage changes. This paper presents a brief primer on calculating and interpreting percentage changes and growth rates. The purpose is to illuminate these measures, facilitate their interpretation, and clarify their usage for economic analysis.

1 Introduction

A strong understanding of calculating and interpreting percentage changes and growth rates is critically important for economists. This is because many fundamental concepts such as the time value of money, and many commonly reported economic measures, such as the rate of return on assets, price inflation, and measures of economic growth, require a firm understanding of percentage changes. Furthermore, economists tend to focus their economic analysis on relative changes of variables of interest rather than absolute changes. Arguably the most important measures in economics are elasticities, which come in many different varieties and represent the percentage change in one economic variable given a one-percent change in another.

This paper presents a brief primer on calculating and interpreting percentage changes and growth rates. The purpose is to illuminate these measures, facilitate their interpretation, and clarify their usage for economic analysis. Four alternative metrics of percentage change are discussed and compared using a simple numerical example. Some parsimonious suggestions are provided for preferring certain measures over others given the nature of the data series under investigation and the purpose of the analysis.

2 Measures of Percentage Change

The analysis of economic data often involves the need to calculate and interpret percentage changes. Some economic data are volatile with large fluctuations between data points, others are more smooth-trending with less variation. The alternative measures of percentage change discussed in this paper can differ substantially depending on the type of economic data series under investigation.

2.1 Fundamental Formulas for Calculating a Percentage Change

As a starting point consider three alternative methods of calculating a total percentage change in a data series, $x_0, x_1, \dots, x_T$. The total percentage change from x_0 to x_T can be calculated on different bases as follows:

$$\textit{Beginning Base:} \qquad \%\Delta x_B = \frac{(x_T - x_0)}{x_0} \tag{1}$$

$$\textit{Average Base:} \qquad \%\Delta x_A = \frac{(x_T - x_0)}{0.5 \times (x_0 + x_T)} \tag{2}$$

$$\textit{Natural Log Base:} \qquad \%\Delta x_L = \ln\left(\frac{x_T}{x_0}\right). \tag{3}$$

Equation (1) uses the beginning period as the base, Equation (2) uses the average base of the beginning and ending periods, and Equation (3) uses the natural logarithm formula. Note that the average base of the beginning and ending periods is a discrete approximation to the continuous analog using natural logarithms, $\%\Delta x_A \cong \%\Delta x_L$. The approximation is good for small changes, but the metrics can be different for large percentage increases as shown in the numerical example that follows.

2.2 Growth Rates

Now consider the discrete compound interest formula that is familiar to economists, relating the present value to the future value of an asset, x. Let i denote the discount rate obtained using discrete compounding.[1]

$$x_T = x_0(1+i)^T \tag{4}$$

Solving for i gives:

$$i = \left[\frac{x_T}{x_0}\right]^{\frac{1}{T}} - 1. \tag{5}$$

We can use Equation (5) to calculate the annual average percentage change (growth rate) in the value of some asset over time. We can also use Equation (5) to calculate the rate of return to some investment opportunity, where x_0 is the present value of the cash outflows, and x_T is the future value of cash inflows. In this case, Equation (5) is called the modified internal rate of return (MIRR) of the investment, which was first introduced in the academic literature in the eighteenth century (Duvillard 1781; Biondi 2006). The more commonly reported measure of the rate of return in the economics literature is the internal rate of return (IRR). Typically, a closed analytical solution does not exist for calculating a conventional IRR (when cash inflows and outflows occur over numerous periods), and numerical methods like interpolation must be used to derive a solution.

The preference of reporting an IRR or a MIRR depends on the application. With an IRR, all of the cash inflows generated over the lifetime of a project are assumed to be reinvested in the project under analysis, which is probably a reasonable assumption when evaluating a targeted investment for a private company. By contrast, the MIRR allows for the incorporation of both an assumed cost-of-debt capital to calculate the present value of cash outflows (x_0 in Equation 5), as well as a potentially alternative reinvestment rate to calculate the future value of cash inflows (x_T in Equation 5). The MIRR is probably more appropriate for evaluating things like public investments in research and development (R&D), which have a cost-of-debt capital that can be pegged to the return on government bonds, and a reinvestment rate that represents a market rate of return.[2]

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[1] Denote the value of an asset at time zero x_0. At discount rate, i, the value of the asset at the end of the first period is, $x_0 + x_0 i = x_0(1+i)$. The value at the end of the second period is, $x_0(1+i) + x_0(1+i)i = x_0(1+i)^2$. The value at the end of period T is, $x_0(1+i)^T$.

[2] There was a recent debate in the agricultural economics literature about the best method to evaluate the economic rate of return to public investments in agricultural R&D. Traditionally, most studies reported an IRR (Alston et al. 2000); however, Alston et al. (2011) and Hurley et al. (2014) argued that the MIRR is a superior measure for evaluating public investments in agricultural R&D. In a comment

Next consider the continuously compounded interest formula using the discount rate, r, and the base of the natural logarithms the mathematical constant, e,

$$x_T = x_0 e^{rT} \tag{6}$$

Solving for r gives:

$$r = \frac{\ln\left[\frac{x_T}{x_0}\right]}{T}. \tag{7}$$

The continuously compounded discount rate is $\%\Delta x_L$ divided by the total number of periods minus one (or data points minus one). We can use Equation (7) to calculate the annual average percentage change in x_t. We can also set the discrete compound interest formula equal to the continuously compounded interest formula to solve for the relationship between the discount rates,

$$(1+i)^T = e^{rT} \tag{8}$$

$$i = e^r - 1. \tag{9}$$

Equation (9) is the commonly used formula for calculating the discrete equivalent to a continuously compounded discount rate. Equations (5) and (7) show that the annual average percentage change in a data series is totally dependent on the choice of endpoints. We denote these the *endpoint* metrics of percentage change. This is not the case in the calculation of growth rates using regression analysis as in the next section, where all of the data points affect the estimated growth rate.

2.3 Trend Analysis

Consider the following specification of the population regression line, where the natural log of the dependent variable x_t is a linear function of a trend variable, t, and a random error term, u_t,

$$\ln x_t = \alpha + \beta t + u_t. \tag{10}$$

The random error terms are independent and identically distributed random variables that follow the normal distribution with conditional expectation equal to zero and constant variance, $u_t \sim N[0, \sigma^2]$. The first-order partial derivative of $\ln x_t$ with respect to the time variable represents the growth rate of x_t,

$$\frac{\partial \ln x}{\partial t} = \beta. \tag{11}$$

The ordinary least squares (OLS) point estimator of the population parameter β is an estimate of the growth rate of x_t and can be compared with an annual average percentage change as described in the previous section. The regression estimate uses all the data points in contrast to the *endpoint* metrics of percentage change. Any large outliers in the data or substantial volatility in the underlying data series can cause large differences in these measures.[3]

3 Numerical Example

to the Hurley et al. (2014) paper, Oehmke (2017) made the case that the IRR is still the preferred measure. Hurley et al. (2017) responded that the MIRR is the superior measure for evaluating public expenditures on R&D.

[3] In the case of the trend regression, additional estimation problems such as autocorrelation may be present that can bias the estimated growth rate.

A simple numerical example is presented to illustrate the concepts covered in the previous section. Table 1 shows two hypothetical data series used in the analysis that follows and the natural logarithms of each series. The endpoints for Data Series (1) and Data Series (2) are the same: $x = 10$ at $t = 1$, and $x = 16$ at $t = 10$. Data Series (1) is a relatively volatile series with large annual fluctuations, and Data Series (2) is a relatively smooth-trending series except for a single large outlier (large decrease in year 8).

Table 1. Two Hypothetical Data Series and Natural Logs

Data series (1)			Data series (2)		
t	x	$\ln x$	t	x	$\ln x$
1	10	2.30	1	10	2.30
2	13	2.56	2	11	2.40
3	9	2.20	3	11	2.40
4	15	2.71	4	12	2.48
5	12	2.48	5	11	2.40
6	16	2.77	6	13	2.56
7	15	2.71	7	14	2.64
8	17	2.83	8	10	2.30
9	22	3.09	9	15	2.71
10	16	2.77	10	16	2.77

Figure 1 shows each of the hypothetical data series, the natural log of each series, and a linear trend added to the natural log series. The figure also includes the estimated linear trend equation and the corresponding R^2 for each data series.

The results from an OLS regression of $\ln x_t$ on a time trend using Data Series (1) are presented below in Equation (12),

$$\ln x_t = \underset{(0.1164)}{2.2642} + \underset{(0.0188)}{0.0690t} \qquad \underset{(s.e.)}{T = 10} \tag{12}$$

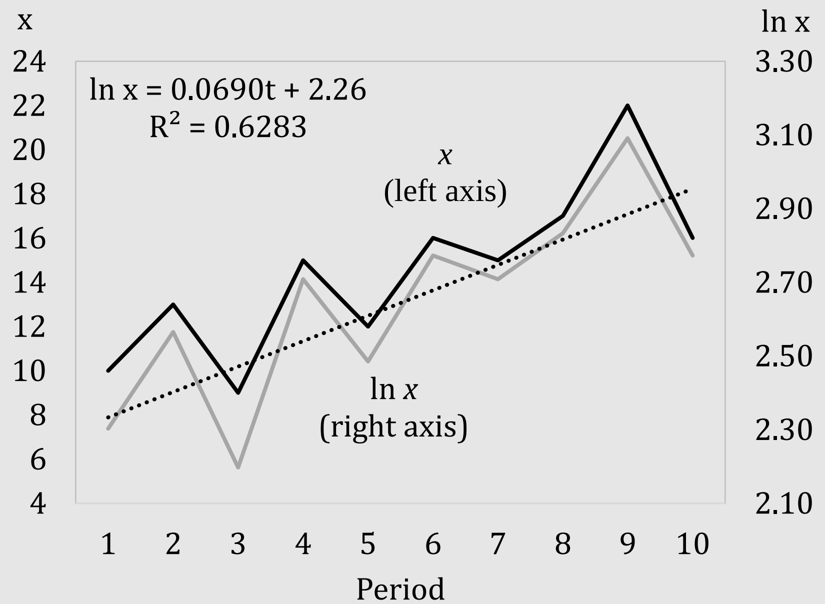
The standard errors are in parentheses. The $R^2 = 0.6283$, and the estimated growth rate $\hat{\beta} = 0.0690$ is statistically significantly different from zero at the 1-percent level of significance. Note that $\hat{\beta}$ represents a continuously compounded discount rate analogous to the previously defined, r, and Equation (9) can be used to convert to a discrete discount rate, i, if this is preferred. In the current application, the equivalent rate under discrete compounding is $i = 0.0714$ or 7.14 percent per period.

Table 2 shows the total percentage change and the growth rate for each metric using Data Series

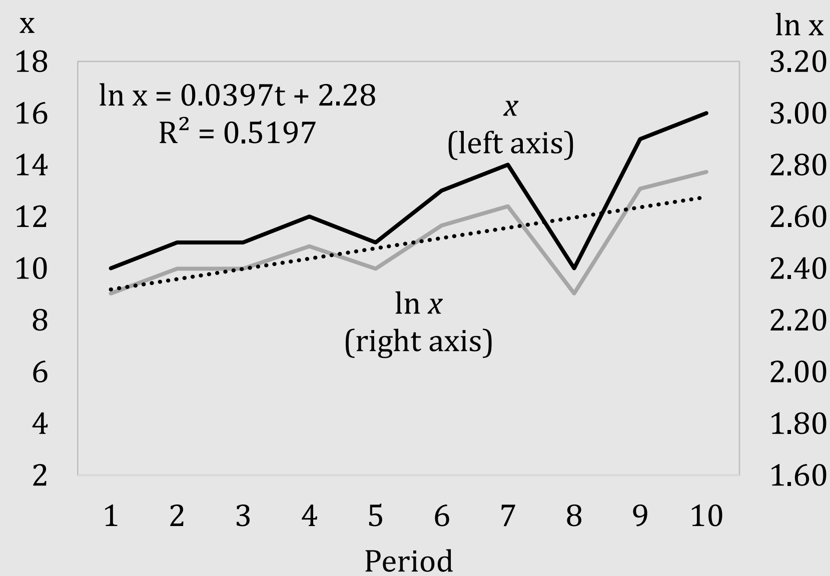
Table 2: Percentage Changes and Growth Rates for Alternative Metrics for Data Series (1)

Panel (a): Endpoint $t = 10$			Panel (b): Endpoint $t = 9$		
Metric	Total	Annual-average	Metric	Total	Annual-average
Beginning base	60.00	6.67	Beginning base	120.00	15.00
Average base	46.15	5.13	Average base	75.00	9.38
Natural log	47.00	5.22	Natural log	78.85	9.86
Regression	62.10	6.90	Regression	67.28	8.41

Note: The beginning, average, and natural log estimates are from Equations (1), (2), and (3). The regression estimates are from Equation (10). The point estimates of total percent change = $\hat{\beta} \times (t - 1)$.



(a) Data Series 1 and Linear Trend



(b) Data Series 2 and Linear Trend

Figure 1. Hypothetical Data with Linear Trend for Natural Log Series

Note: The figures include the linear trend equation and the R^2. The estimated growth rate from the linear trend regression in Data Series (1) is 6.90 percent, which is statistically significantly different from zero at the 1-percent level of significance. In Data Series (2), the estimated growth rate is 3.97 percent, which is statistically significantly different from zero at the 5-percent level of significance.

(1) and two different endpoints for the analysis, t = 9 and t = 10. The first three rows of each panel in Table 2 show the calculated percentage changes using Equations (1), (2), and (3), and their corresponding growth rates.

The final row of each panel is the point estimator of the growth rate from Equation (10). In Table 2 Panel (a), when t = 10, the three *endpoint* metrics of the growth rate range from 5.13 to 6.67 percent. In Panel (b), when t = 9, the *endpoint* metrics range from 9.38 to 15.00 percent. This is an example of the sensitivity of standard measures of percentage change to the choice of endpoint, and the potential for "cherry-picking" a desired result. Also note that the discrete approximation to the natural log formula is accurate for modest increases in the data series x_t as in Panel (a), but the two measures diverge for large increases as in Panel (b). If we report the regression estimates of the growth rate for both endpoints (t = 9 and t =10), which use all of the data, the estimate does not change as drastically falling from 8.41 to 6.90 percent. This is a case where the regression estimate of the growth rate has a clear advantage over the alternatives. When working with volatile data series, the choice of the range of data under analysis is critically important, especially when using metrics that depend solely on the endpoints to calculate the growth rate.

Next, we turn to hypothetical Data Series (2) in Table 1 and Figure 1, to illustrate the effects of a substantial outlier in the data series. Recall the endpoints of each of the Data Series (1) and (2) are the same, x = 10 at t = 0 and x = 16 at t = 10; therefore, the first three metrics of the percentage change and the growth rate (the beginning period base, average base, and natural log) are the same as in Table 2 Panel (a). Only the regression estimate of the growth rate differs between the two data series. In Data Series (1), the point estimate of the growth rate is 6.90 percent, and in Data Series (2) the estimate falls to 3.97 percent. The large single outlier in Data Series (2) caused a dramatic reduction in the point estimate of the growth rate. The regression estimate of the growth rate is sensitive to large outliers in the data, and this is an important cautionary note when using and interpreting these measures.

To summarize, be cautious when reporting standard measures of percentage change such as those calculated using the beginning period base or the natural logarithm formula when the choice of endpoints is important. This is especially true when analyzing data series that vary significantly from year to year as in hypothetical Data Series (1). In this case, a regression estimate is likely the optimal option. Conversely, if the data series is mostly smooth-trending, but has a significant outlier like in Data Series (2), a regression estimate of the growth rate will be substantially impacted.[4] In this case the standard measures are superior. These results should serve as a cautionary tale about the importance of understanding measures of percentage change, and in particular which method was used to construct the measures.

4 Conclusion

Important economic measures such as inflation rates, elasticities, the rate of return on assets, and interest rates, represent the percentage change of some underlying economic data series. Are the measures total percentage changes, annual averages, or regression estimates, and which method was used to calculate them? Do the metrics represent continuous compounding or discrete compounding? As an economic practitioner, the author suggests the natural log formula (Equation 3) to calculate a total percentage change, as well as an estimate of the annual average after dividing by the number of periods minus one; however, in addition, a regression estimate of the growth rate is sometimes obtained. The difference between the annual average and the regression estimate of the growth rate will be large when the data are volatile or there are significant outliers in the data. A good rule of thumb is to use a standard measure of percentage change like a natural log formula when the data series under analysis is relatively smooth-trending, the data series has a large outlier, or the choice of endpoints is not of particular importance. The regression estimate of the growth rate is superior when the data are volatile and the choice of endpoints is important to the analysis.

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[4] It is also possible to use a dummy variable for a large outlier year if there is a justification.

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